

Incubation project report:  
**Computational Topology for Exploring  
Time-varying Volume Data**

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# Collaborators

## **Students**

- Lorenzo Ibarria, Byung-Moon Kim (Georgia Tech)
- Ajith Mascarenhas, Martin Isenburg (UNC)

## **Faculty/Researchers**

- Peter Lindstrom, Valerio Pascucci (LLNL)
- Andrzej Szymczak (Georgia Tech)
- Herbert Edelsbrunner, John Harer, (Duke U.)

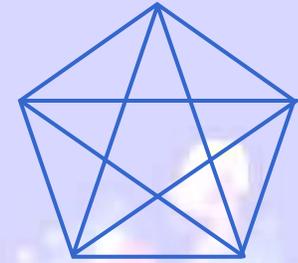
# Focus on huge 4D scalar fields

**Input:**  $P(x,y,z,t)$  sampled on regular 4D grid

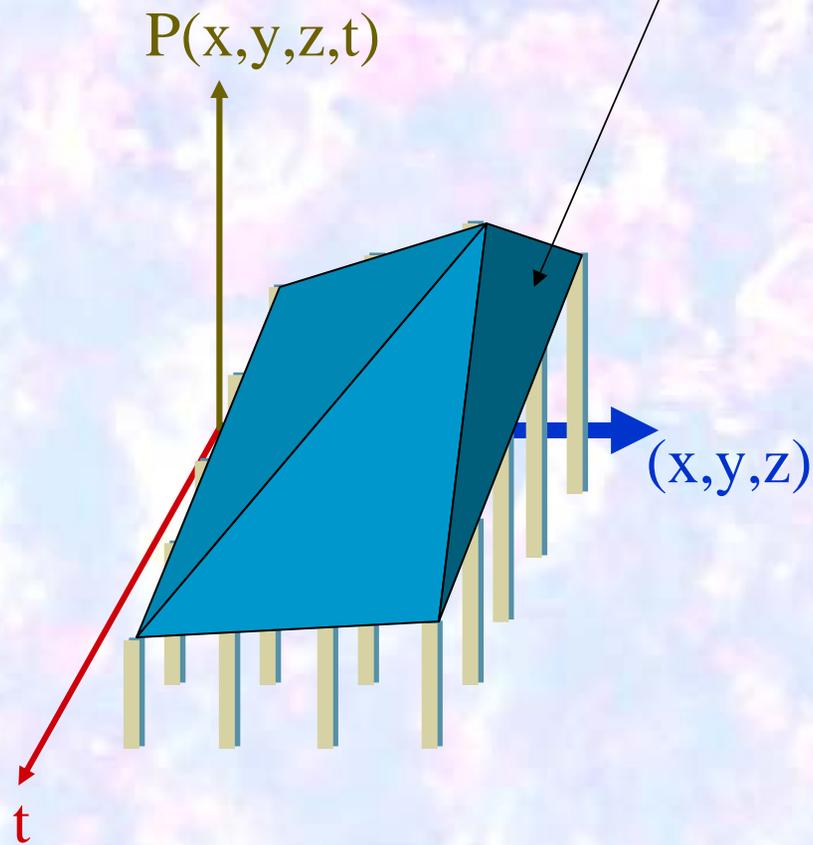
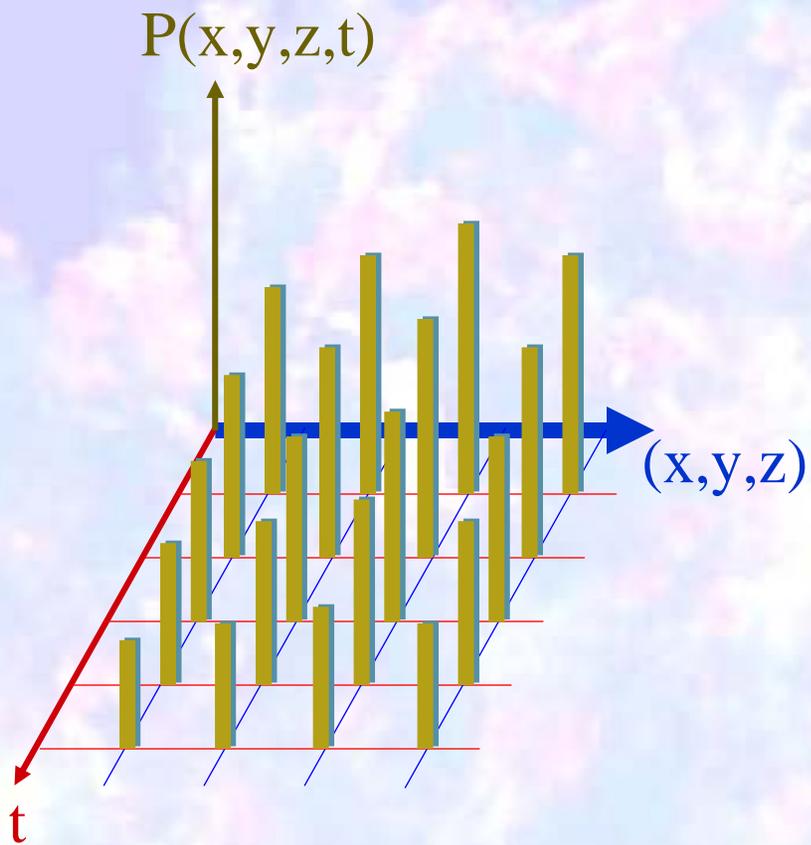
## OUTLINE

- Out-of-core compression/decompression
- Adaptive simplification of penta-mesh
- Iso-surface extraction  $\{(x,y,z):P(x,y,z,t)=p\}$
- Iso-surface compression/decompression
- Interactive exploration

# Regular... or not?



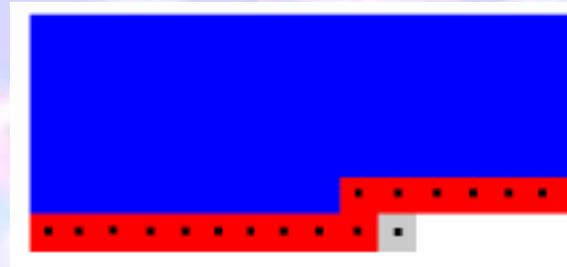
pentatope in 5D



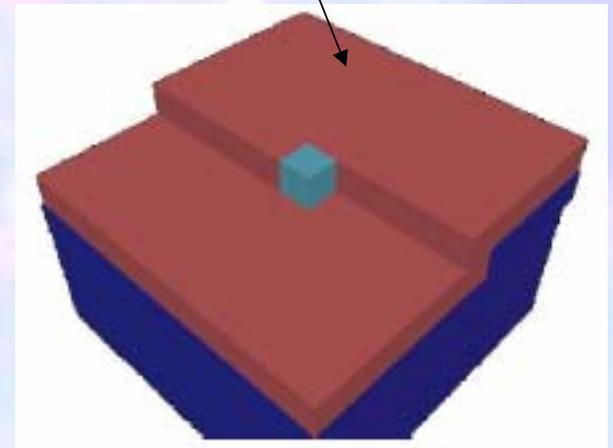
# Lossless compression

- **Out-of-core compression/decompression of large n-dimensional scalar fields**
  - Lawrence Ibarria (Georgia Tech)
  - Peter Lindstrom (LLNL)
  - Jarek Rossignac (Georgia Tech)
  - Andrzej Szymczak (Georgia Tech)
- To appear in Proc. of Eurographics'03

# Scanline codecs



foot-print  $\approx 1$  slice



For  $t=0$  to  $t_{\max}$  do

For  $z=0$  to  $z_{\max}$  do

For  $y=0$  to  $y_{\max}$  do

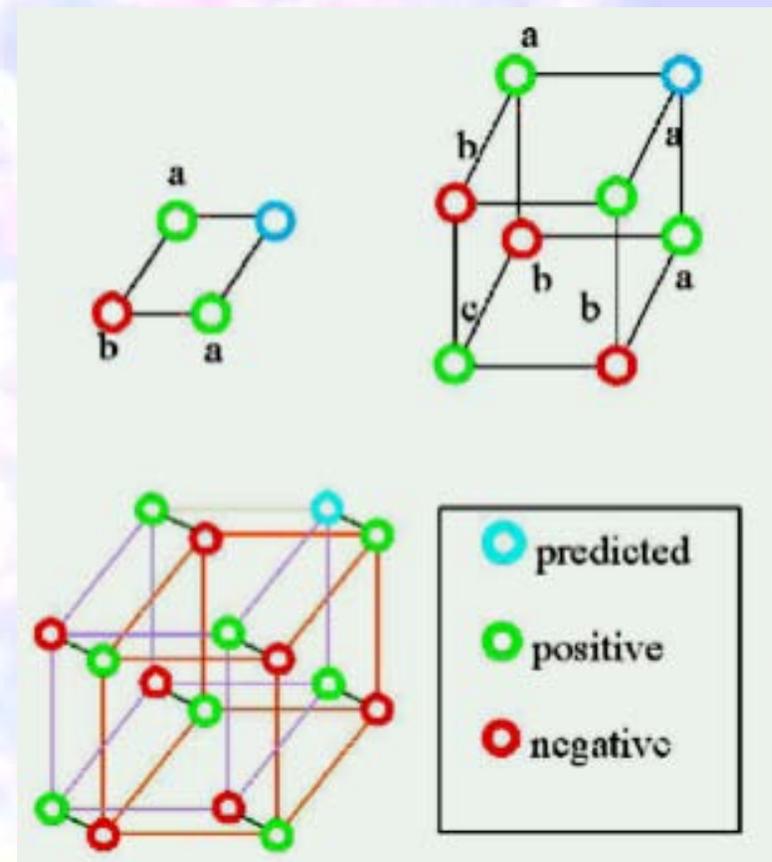
For  $x=0$  to  $x_{\max}$  do {

Predict  $P(x,y,z,t)$  from visited neighbors

Encode/decode the correction}

# Lorenzo predictor

- Predict P at corner of hypercube from values at the other corners
  - Set origin at opposite corner
- 2D:  $P(1,1)=P(1,0)+P(0,1)-P(0,0)$
- 3D:  $P(1,1,1)=\sum a-\sum b+c$
- 4D:  $P(1,1,1,1)=\sum n_1-\sum n_2+\sum n_3-n_4$ 
  - $n_i$  is reachable through  $i$  edges
  - Exact predictor for cubics in 4D

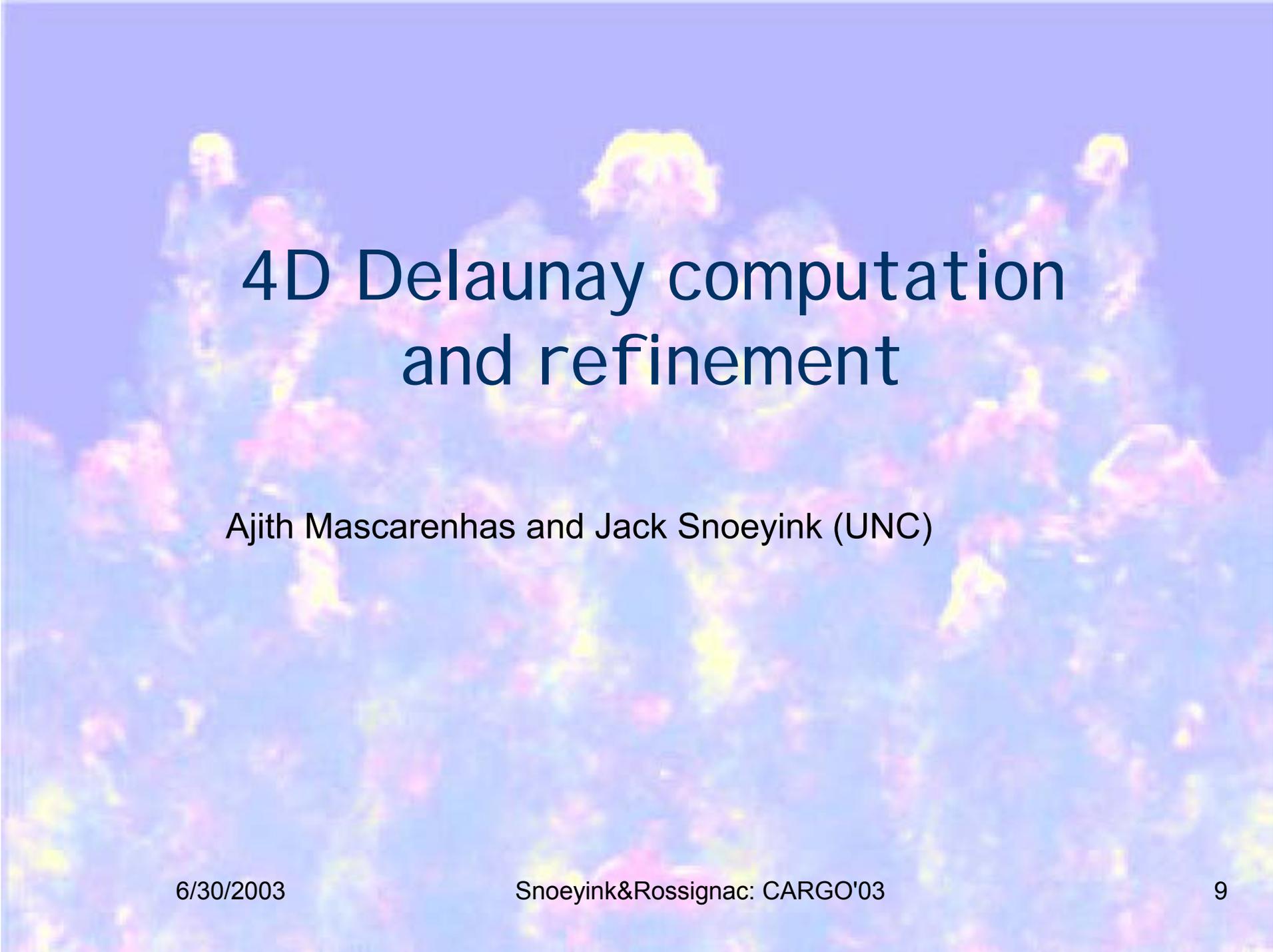


# Results for lossless compression

- Small memory footprint
- Easy to implement
- Exactly reconstructs polynomials of degree  $n-1$
- Can outperform wavelets

DataSet	4D Lorenzo Predictor	Cubic Wavelets
Smooth $64^4$	0.16 Bits/Symbol	0.20 Bits/Symbol
Harsh $64^4$	3.73 Bits/Symbol	3.28 Bits/Symbol
Harsh $128^4$	1.75 Bits/Symbol	1.80 Bits/Symbol

**Table 1:** *This shows the entropy of the residuals computed with wavelets and the Lorenzo predictor for a 4D dataset, in a lossless fashion.*

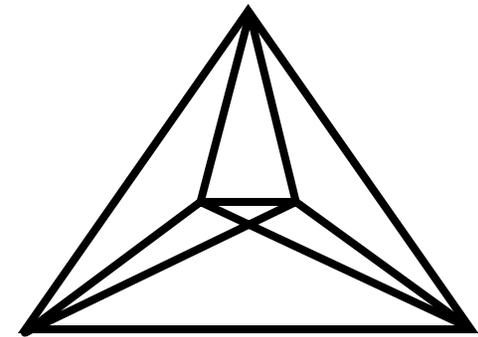
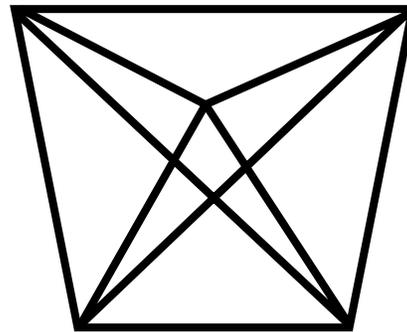
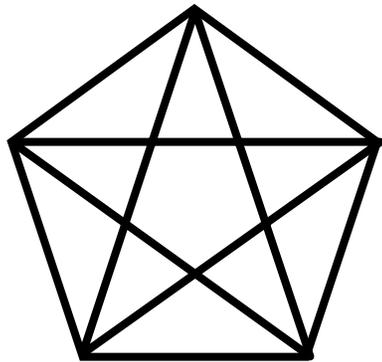


# 4D Delaunay computation and refinement

Ajith Mascarenhas and Jack Snoeyink (UNC)

# Isosurfaces from pentatopes

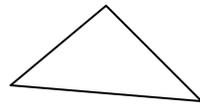
- Mesh of 5-simplices are 4d surface in 5d:
  - Pentatopes (4d)
  - Tetrahedra (3d)
  - Triangles (2d)
- Slice away 2 dimens to form isosurface:
  - Faces (2d)
  - Edges (1d)
  - Vertices (0d)



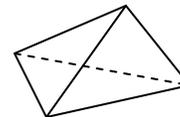
# Yes, this is crazy...

- Regular grid stores **values** only
- mesh adds (x,y,z,t) & incidence

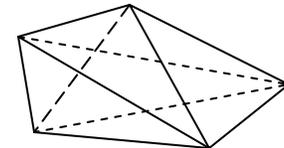
2d: Triangle



3d: Tetrahedron



4d: Pentatope



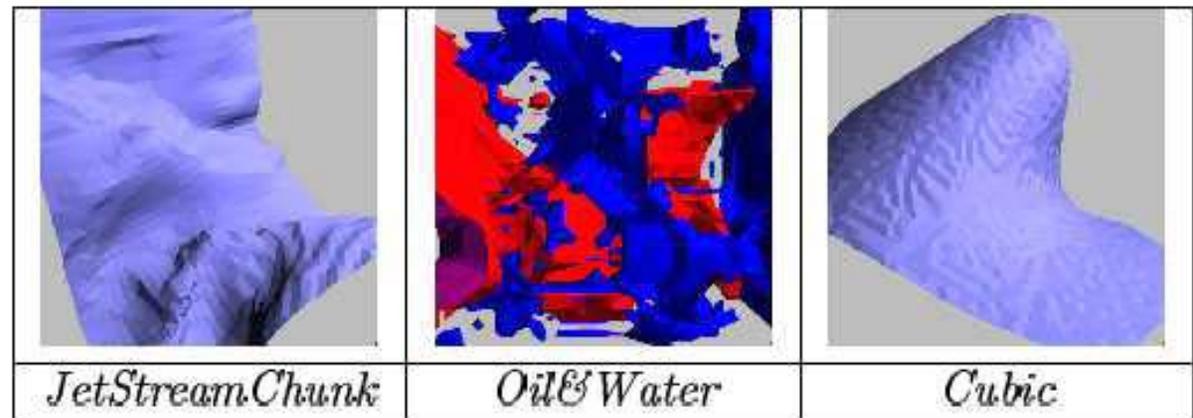
Simp/vertex	2	7	28
Words/vertex	15	51	244

- Allows local refinement, so we try anyway...

# Incremental 4D Delaunay

- Can't use simplification (full resolution mesh is too large)
- Implemented insertion heuristic to build interpolating mesh
  - Add points with greatest error to 4D Delaunay and retriangulate

As expected:  
Mesh requires  
large storage  
space

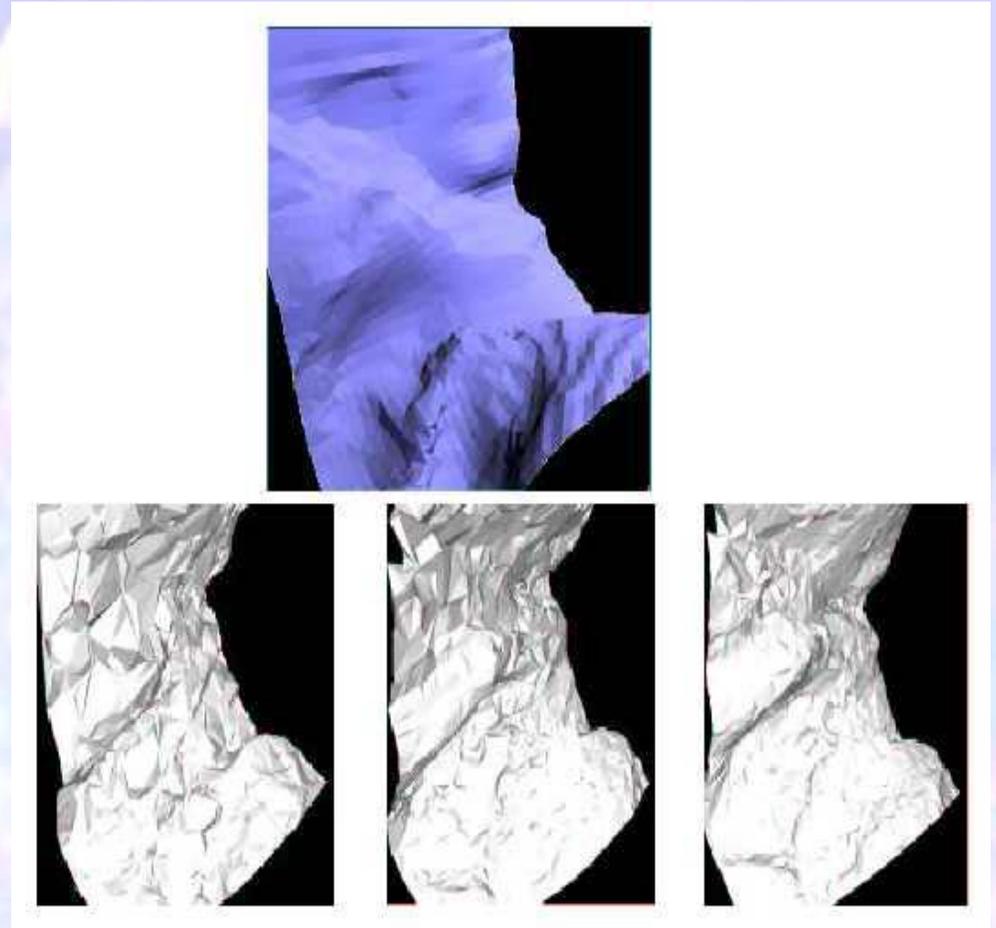


Data-set	Data-set size(bytes)	# vertices	# pentatopes	Pentamesh size(bytes)	mesh/data-set size.
<i>JetStreamChunk</i>	1,048,576	72,035	1,719,999	69,160,135	65.96
<i>Oil&amp;Water</i>	280,000	19,018	435,771	17,525,930	62.6
<i>Cubic</i>	2,825,761	17,271	430,883	17,321,675	6.13

# Results

- Severe aliasing for sections of low-resolution penta-meshes
- Iso-surface from the *JetDataChunk* data-set at iso-value = 128, time = 12

Iso-surfaces at three stages of refinement



vertices:	16K	45K	72K
pentatopes:	0.4M	1.0M	1.7M

# Sphere-based Delaunay

- The fast Delaunay implementation is interesting:
  - Sphere-based
  - Exact, handling all degeneracies
  - To be released this summer.



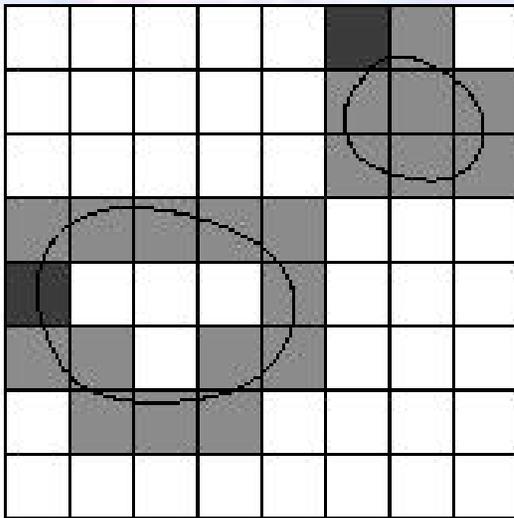
# Seed Set Computation for Iso-surface Extraction in Time-varying Volumetric Data

student:

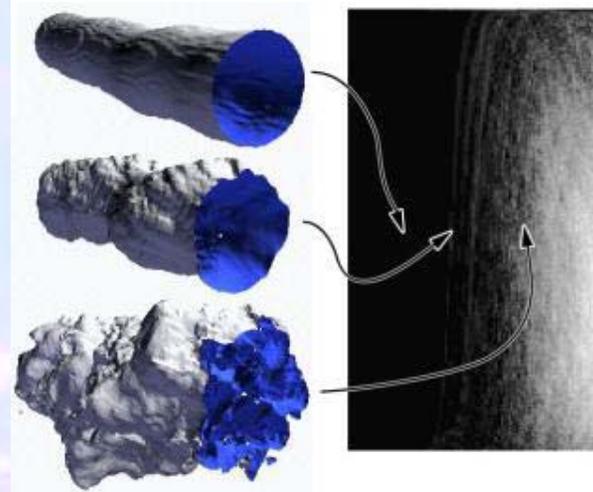
Ajith Mascarenhas (UNC)

# Extract iso-surface component

- Volume data is often visualized by extracting iso-surfaces.
- Efficient iso-surface extraction.
  - Start at cell known to intersect iso-surface - seed cell.
  - Extract iso-surface component by visiting neighboring cells recursively



■ Seed cells    ■ Intersecting cells



Iso-surfaces of Jet Stream simulation data.

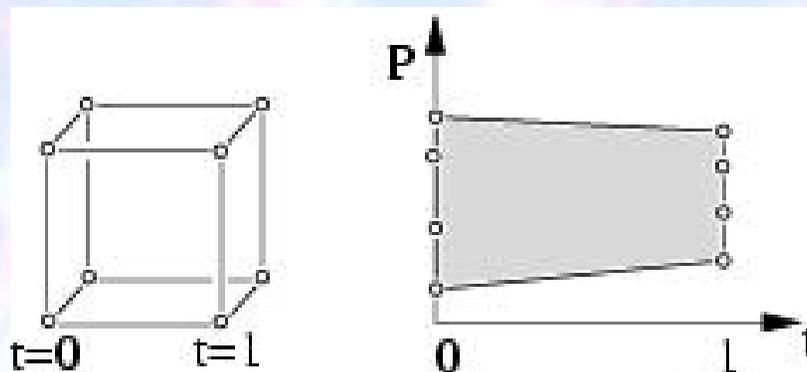
# Seed set computation

## – AIM

- Compute seed set for time-varying volumetric data (4-d).

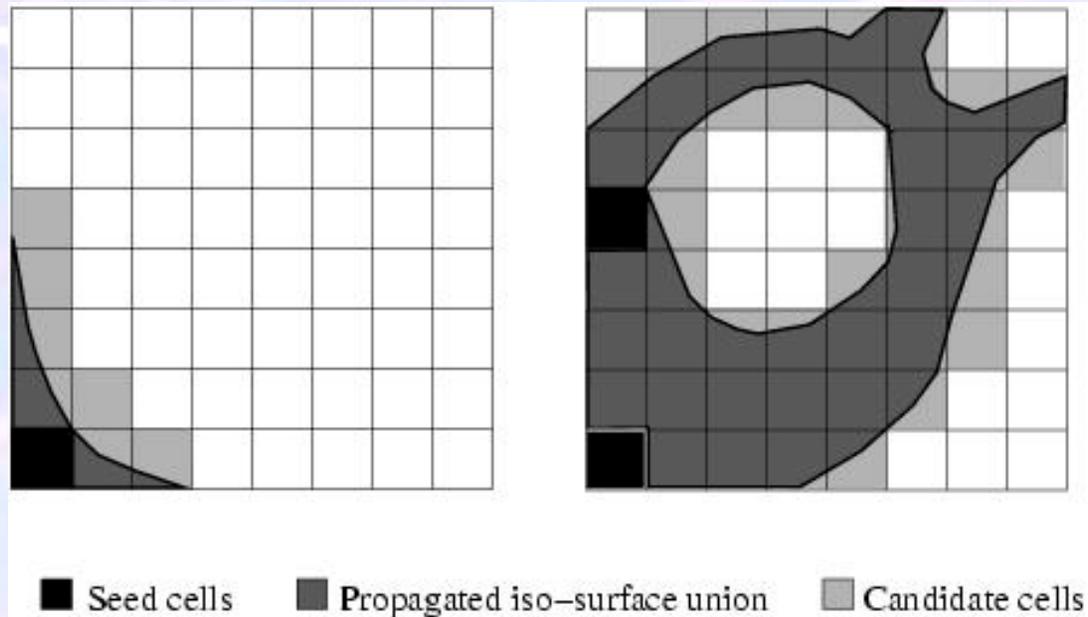
## – Range of a cell.

- Set of iso-values for which the cell intersects an iso-surface.
- For volumetric data in 3D, range is an interval on the real line.
- For time-varying volumetric data, the iso-value is a *(Value, time)* pair => the range is a convex set on the plane.



# Seed set computation in 3-d

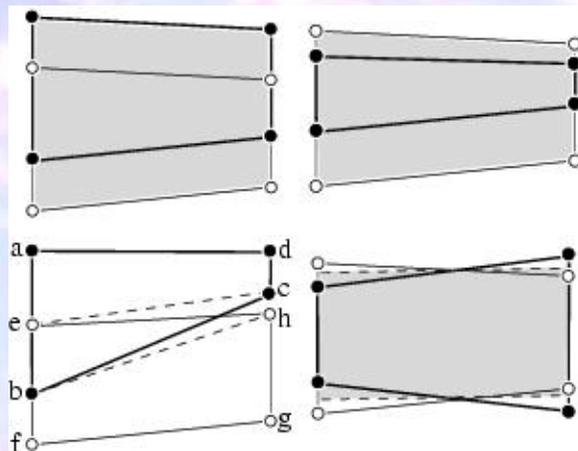
- Greedy Algorithm (Bajaj et al.)
  - Propagate union of iso-surfaces starting from arbitrary cell.
  - While more seeds can be chosen
    - Choose candidate cell with largest uncovered range as next seed.
    - Propagate residual range.



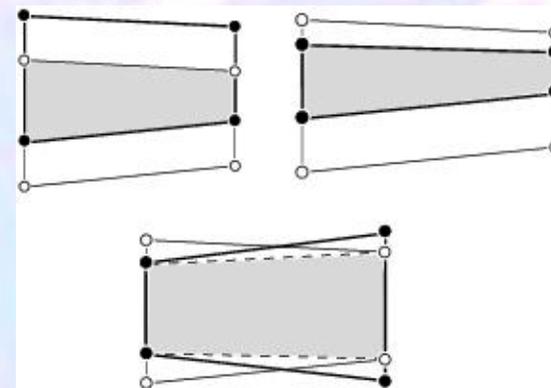
# Range Operations in 4-d

- Union, Intersection, Difference

- Exact for some cases.
- Approximation for others.



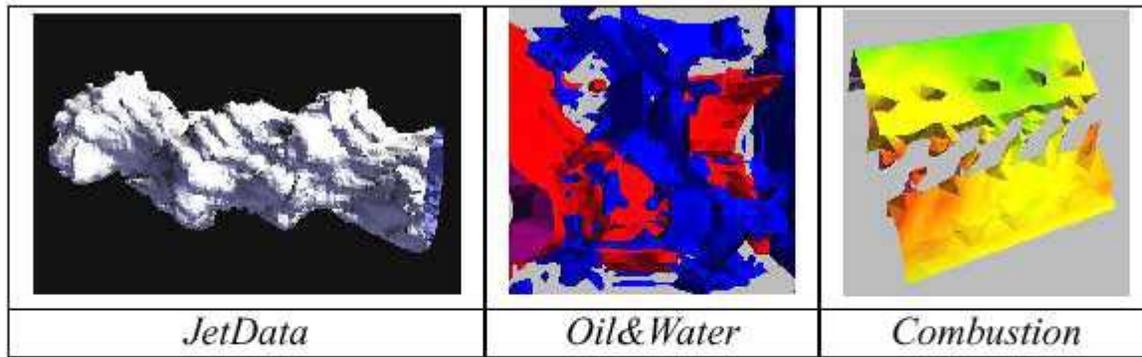
Union



Intersection

# Seed set computation in 4-d

## – Results



**Table 1. Iso-surfaces of Data-sets**

Data-set	Dimensions( $x \times y \times z \times t$ )	no. cells	seed-set size	% seed-set./cells
<i>JetData_4</i>	$64 \times 64 \times 64 \times 100$	24,754,653	57,928	0.23
<i>Oil&amp;Water</i>	$28 \times 25 \times 25 \times 16$	233,280	2,814	1.2
<i>Combustion</i>	$27 \times 16 \times 20 \times 20$	140,790	10,544	7.4

# Iso-surface extraction from 4D regular grids

- Time-varying Reeb graphs, Jacobi sets, & the  $\gamma$ -curve

- Jack Snoeyink and Ajith Mascarenhas (UNC)

- Joint work with:

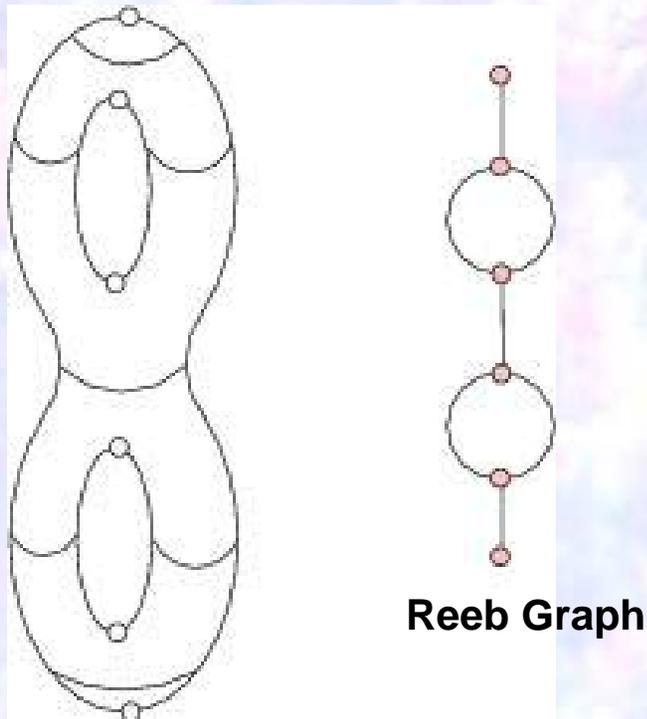
- Herbert Edelsbrunner, John Harer, (Duke U.)

- Valerio Pascucci, (LLNL)

# Contour trees

Reeb Graph captures topological changes of iso-contours.

- A Contour Tree is a special kind of Reeb graph.

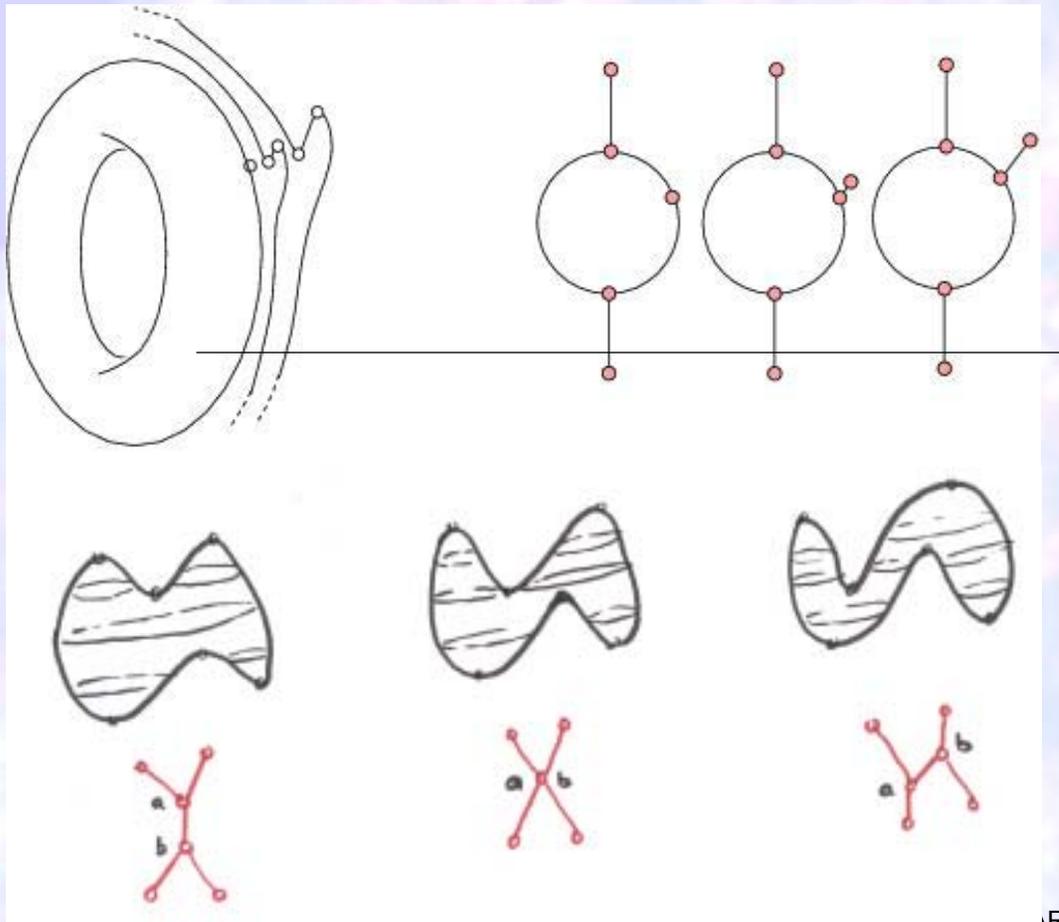


Reeb Graph

- Critical pts. of function map to nodes of Reeb graph
- AIM: Study changes to Reeb graph as function changes with time.
- Construct time-varying Contour Trees for time-varying volumetric data.

# Evolution of contour trees?

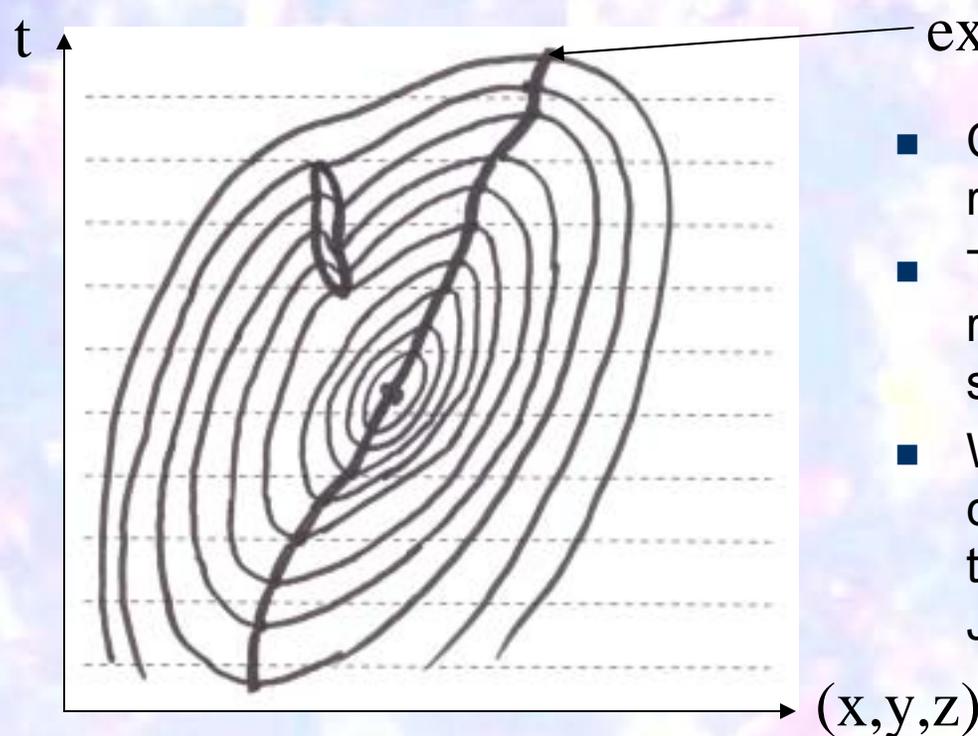
Classified Reeb Graph changes as a function of time



- **Contraction** of arc in Reeb graph = destruction of a pair of critical points.
- **Anti-Contraction** of arc in Reeb graph = creation of pair of critical points.
- **Critical point swap:** A pair of critical points change their order in the function value.
- **Complexity analysis:** Not suitable for large, noisy data sets...

# Jacobi sets

- Jacobi sets: Simultaneous critical points of several Morse functions.
- Example with two functions  $f, g$ .
  - Jacobi set is a 1-manifold here.



extremum of  $P(x,y,z,t)=p$

- Consider level set of  $g$ , and restrict  $f$  to the level set.
- The critical points of the restriction belong to the Jacobi set.
- When the level varies continuously the critical points of the restriction sweep out the Jacobi set.

# Jacobi Sets

- Used in studying time-varying Reeb graph.
  - Critical points map to nodes of Reeb graph.
  - Time-trace of critical points used to understand changes to Reeb graph.
  - Jacobi Set gives the time-trace of critical points.
- Software to compute Jacobi sets of time-varying volumetric data developed by Ajith Mascarenhas (UNC) at LLNL in Summer 02.
  - Tested on moderate data-sets.
  - Being extended to compute time-varying contour tree
  - Requires simplification for large data sets

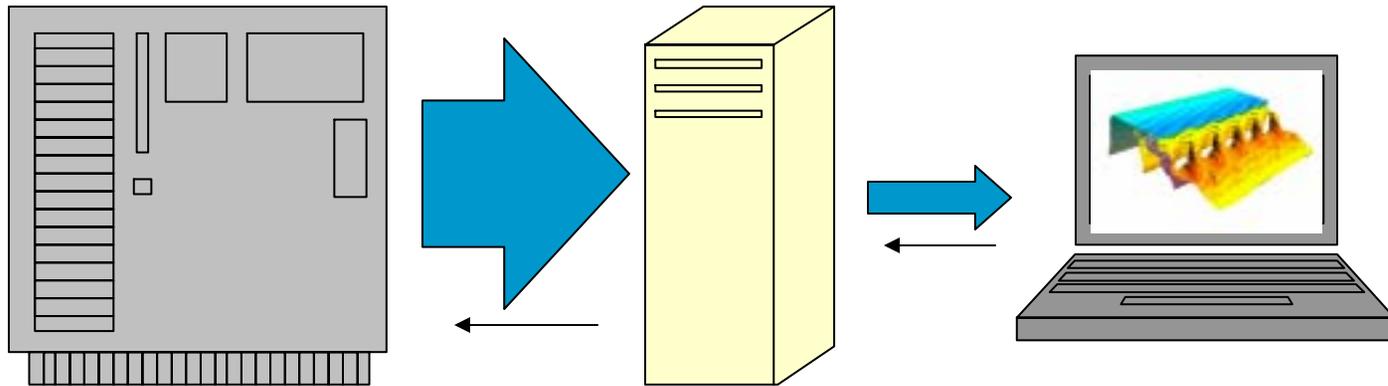
# Iso-surface compression

- Out-of-core compression and simplification of iso-surfaces and geometric models
- Jack Snoeyink, Martin Isenburg (UNC)
- Based on segmentation

⇒ Part of a 6GB iso-surface that compresses to 640MB losslessly, & decompresses with a memory footprint of 9MB.



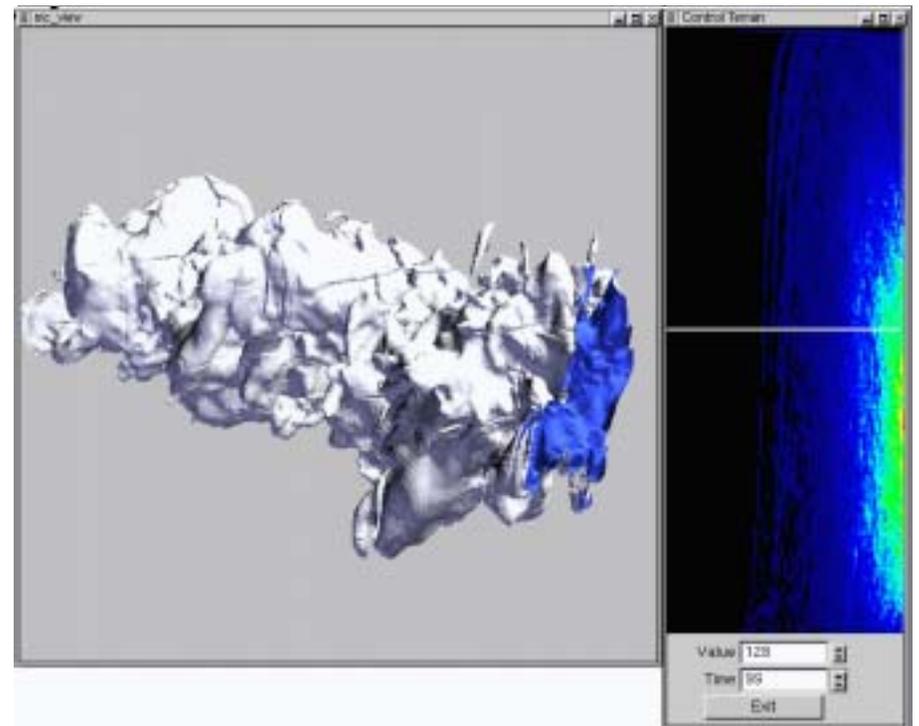
# Tools for remote exploration



What tools can we provide to suggest  
when & where to look  
in large simulation data sets?

# Safari on the (p,t) plane

- Samples from pressure  $p = f(x, y, z, t)$ .
- Level sets defined by two parameters:  
$$L(P, T) = \{ (x, y, z) : P = f(x, y, z, T) \}$$
- Partition the data dimensions
  - Viewing volume  $(x, y, z)$
  - Control plane  $(p, t)$



# How it all will come together

- Use Lorenzo predictor to compress data on server for archival and posting
- Use contour trees analysis in 4D on server to compute image for  $(p,t)$  terrain
- Use Safari on client to plan, conduct, record, annotate exploration on the  $(p,t)$  terrain
- Use seed-set on server to quickly extract iso-surface
- Use out-of-core compression/simplification to transmit desired iso-surface
- Use Lorenzo predictor decompression to download selected portions of the original 4D data for local exploration/analysis

# Publications

## ■ Appeared/Accepted

- L. Kettner, J. Rossignac, J. Snoeyink, The Safari Interface for Visualizing Time-dependent Volume Data Using Iso-Surfaces and a Control Plane, CGTA 25:1-2(2003), pages 97-116
- H. Carr, J. Snoeyink, Path Seeds and Flexible Isosurfaces: Using Topology for Exploratory Visualization, IEEE/EG VisSym'03.
- L. Ibarria, P. Lindstrom, J. Rossignac, A. Szymczak, Out-of-core compression and decompression of large n-dimensional scalar fields, Eurographics 2003.

## ■ Submitted

- M. Isenburg, P. Lindstrom, S. Gumhold, J. Snoeyink, Large Mesh Simplification using Processing Sequences, IEEE Vis 03.

## ■ In preparation

- A. Mascarenhas, Y. Liu, J. Snoeyink, Sphere-based Implementation of Delaunay Diagram Computation
- A. Mascarenhas, J. Snoeyink, Seed Set Computation for Isosurface Extraction in Time-varying Volumetric Data

# Conclusions of incubator project

- New solutions for out-of-core codec are effective
  - Lorenzo predictor for 4D regular grids
  - Segmentation of iso-surface for small foot print
- New methods for extracting/analysing iso-surfaces are elegant and promising
  - Time-varying contour tree
  - Greedy seed set computation in 4D
- Adaptive refinement does not look promising
  - Pentatope meshes in 4D use lots of storage
  - Delaunay refinement is expensive and difficult
  - Isosurfaces from slices of pentas show artifacts

# Thanks to

- Students
  - Lawrence Ibarria (GATech)
  - Ajith Mascarenhas, Martin Isenburg (UNC)
- Lawrence Livermore National Labs
  - Valerio Pascucci
  - Peter Lindstrom
- Other Collaborators
  - Andrzej Szymczak (GATech)
  - Herbert Edelsbrunner, John Harer (Duke)
  - Lutz Kettner (MPI Saarbruecken)
- NSF/DARPA CARGO